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INVERSE MEAN FREE PATH, STOPPING POWER, CSDA RANGE, AND STRAGGL--ETC(U)  
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AND STRAGGLING IN GE AND GAAs FOR ELECTRONS OF  
ENERGY LESS THAN OR  $\approx$  10 KEV

OAK RIDGE NATIONAL LABORATORY, TENNESSEE

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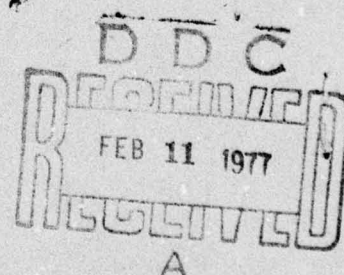
INVERSE MEAN FREE PATH, STOPPING POWER, CSDA RANGE,  
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ENERGY  $\leq 10$  keV

Oak Ridge National Laboratory

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## CONTENTS

I.	INTRODUCTION.....	4
II.	GENERAL FORMULATIONS.....	5
III.	DIMFP's (DIFFERENTIAL INVERSE MEAN FREE PATHS) FOR THE VALENCE BAND ELECTRONS.....	8
IV.	DIMFP's FOR INNER SHELLS.....	12
V.	EXCHANGE CORRECTED DIMFP's AND FORMULAE FOR THE TABULATIONS.....	17
VI.	REFERENCES.....	20
VII.	GERMANIUM: EXPLANATION OF TABLES.....	22
	TABLE 1A — INVERSE MEAN FREE PATH OF ELECTRONS IN GERMANIUM.....	24
	TABLE 1B — STOPPING POWER OF GERMANIUM FOR ELECTRONS....	25
	TABLE 1C — CSDA RANGE AND STRAGGLING OF ELECTRONS IN GERMANIUM.....	26
VIII.	GALLIUM ARSENIDE: EXPLANATION OF TABLES.....	27
	TABLE 2A — INVERSE MEAN FREE PATH OF ELECTRONS IN GaAs.....	29
	TABLE 2B — STOPPING POWER OF GaAs FOR ELECTRONS.....	30
	TABLE 2C — CSDA RANGE AND STRAGGLING OF ELECTRONS IN GaAs.....	31
IX.	TABLE 3 — UNIVERSAL FUNCTIONS EMPLOYED IN THE EVALUATION OF INNER SHELL CONTRIBUTIONS TO INVERSE MEAN FREE PATH, STOPPING POWER, AND MEAN SQUARE ENERGY LOSS.....	32

## I. INTRODUCTION

A quantitative description of the interaction of electrons with matter over a large range of energies is a subject of basic importance in a wide variety of theoretical and applied areas. From the theoretical standpoint, calculations of energy loss and range of electrons in many different materials have formed the basis of at least two extensive tabulations.<sup>1,2</sup> Both of these works are restricted to electron energies  $\geq 10$  keV and are based on the Bethe theory of stopping power including various modifications and corrections (e.g., density-effect corrections). We feel that similar tabulations for electron energies  $\leq 10$  keV, based on a priori calculations using currently available theoretical information, will provide useful guides for interpretation of experimental data as well as input for calculations in applied areas. Our earlier tabulations for the electron energy region  $E \leq 10$  keV include the solids Al and  $\text{Al}_2\text{O}_3$ ,<sup>3</sup> Si and  $\text{SiO}_2$ ,<sup>4</sup> and Ni, Cu, Ag, and Au.<sup>5</sup> Although some of the theoretical framework required for the calculations presented here for Ge and GaAs is identical to that in Refs. 3 and 4, it will be restated here for convenience of the users of this tabulation.

Model calculations are used in the work here to describe the valence bands of the solids. The differential inverse mean free path (DIMFP), which forms the basic function required in our work, will be derived from a model insulator theory<sup>3,4</sup> applied to the valence bands in Ge and GaAs. The more tightly bound, inner shells of the atoms in the solids will be assumed to be essentially unchanged in



character from those in free atoms and the DIMFP for excitation of electrons from the inner shells will be based on classical binary collision (CBC) theory.<sup>6,7,8</sup> An extension of CBC theory in which the cross sections are constrained to obey certain sum rules has been applied in the study of the electron slowing-down spectrum in Si.<sup>9</sup> Given the DIMFP's associated with the most important electron interaction processes in the solids, we then calculate inverse mean free paths, stopping powers, csda ranges, and range and energy straggling for electron energies from a few eV to 10 keV.

The details of the components of our calculations are described more fully in the next three sections. In Section V exchange corrections are discussed, expressions given for the exchange corrected DIMFP's, and formulas used in these tabulations are displayed.

## II. GENERAL FORMULATIONS

A charged particle passing through a solid interacts with a large number of electrons simultaneously, and it is thus appropriate to speak of a mean free path of the charged particle against energy loss to the solid. Assuming the effect of the charged particle on the medium may be treated in first Born approximation, the inverse mean free path, differential in momentum transfer,  $\hbar\vec{k}$ , and energy transfer,  $\hbar\omega$ , for a particle of velocity  $v$  is given by

$$\frac{d^2\mu}{dkd\omega} = \frac{2e^2}{\pi\hbar v^2} \frac{1}{k} \text{Im} \left[ \frac{-1}{\epsilon(k,\omega)} \right] \quad (1)$$

where  $\epsilon(k, \omega)$  is the exact dielectric function of the solid.<sup>10</sup> We assume in this work that the solid is isotropic and homogeneous.

For our calculations of inverse mean free path, stopping power, etc., it is sufficient to compute inverse mean free paths differential in energy transfer only. This differential inverse mean free path (DIMFP) for energy loss  $\hbar\omega$  by an electron with energy  $E = mv^2/2$  in the solid is given by

$$\tau(E, \hbar\omega) \equiv \frac{d\mu}{d(\hbar\omega)} = \frac{1}{\pi a_0 E} \int_{k_-}^{k_+} \frac{dk}{k} \operatorname{Im} \left[ \frac{-1}{\epsilon(k, \omega)} \right] \quad (2)$$

where  $\hbar k_{\pm} \equiv \sqrt{2m} \left[ \sqrt{E} \pm \sqrt{E - \hbar\omega} \right]$  and  $a_0 \equiv \hbar^2/me^2$ . This expression assumes that the energy-momentum relation for a swift electron in the solid does not differ appreciably from that of a free electron in vacuum.

Given  $\epsilon(k, \omega)$  for the solid, the quantities of interest here follow directly from  $\tau(E, \hbar\omega)$ . The inverse mean free path of the electron,  $\mu$ , is given by integrating over allowed energy transfers as

$$\mu(E) = \int d(\hbar\omega) \tau(E, \hbar\omega) . \quad (3)$$

The rate of energy loss of the electron, or the stopping power of the medium, is given by

$$S(E) \equiv -dE/dx = \int d(\hbar\omega) \hbar\omega \tau(E, \hbar\omega) , \quad (4)$$

and the mean square energy loss per unit path length by

$$\Omega^2(E) \equiv \int d(\hbar\omega) (\hbar\omega)^2 \tau(E, \hbar\omega) . \quad (5)$$

With these results we may calculate the range of an electron in the continuous slowing-down approximation (csda range) by

$$R_o(E) = \int_{E_o}^E dE' / S(E') \quad (6)$$

The lower limit on this integration will be discussed further in Section V. The mean square fluctuation in the range or "range straggling" will be calculated from Eq. (5) and Eq. (4) as<sup>11</sup>

$$(R - R_o)_{AV}^2 \equiv \int_{E_o}^E dE' \Omega^2(E') / [S(E')]^3 . \quad (7)$$

In practice, the DIMFP will be evaluated as a sum of contributions from various distinct processes. For example, we calculate a DIMFP for removing an electron from inner shells and a DIMFP for interaction with electrons in valence bands. The total DIMFP used to describe the interaction of an electron with the given solid will be given by

$$\tau(E, \omega) = \sum_i \tau_i(E, \omega) \quad (8)$$



where the sum over  $i$  adds the contributions from the various interaction processes. The evaluation of the  $\tau_i$ 's for Ge and GaAs is described in the next two sections.

### III. DIMFP's FOR VALENCE BAND ELECTRONS

The model which we have developed to describe the dielectric response function of an insulator is related to that employed by Fry<sup>12</sup> in which the ground state wave function of the valence electrons is described in the tight-binding approximation, while excited states are represented by orthogonalized plane waves (OPW). In our use of the model to obtain a dielectric response function we fix the normalization of the OPW excited states by requiring that the sum rule  $\int_0^\infty d\omega \omega \text{Im}[\epsilon(k, \omega)] = 2\pi^2 n e^2 / m$  be obeyed where  $n$  is the density of electrons in the valence band. In addition we assume that the solid is uniform and homogeneous. The dielectric response function corresponding to this model solid is convenient and flexible for use, can be fitted to the optical dielectric function in the limit of very long wavelengths ( $k \rightarrow 0$ ), and describes the single-particle properties of excited electrons. The existence of plasma oscillations emerges naturally as one studies the response of the system to longitudinal electric perturbations. We have found quite reasonable results in the application of this model to  $\text{Al}_2\text{O}_3$  and  $\text{SiO}_2$ .<sup>3,4,13,14</sup>

Since a detailed discussion of the insulator model is planned for publication,<sup>15</sup> we quote here only the results needed for these

calculations. The result required here is the imaginary part of the dielectric response function for the model insulator given by

$$\text{Im}[\epsilon(k, \omega)] = \pi n e^2 \Gamma / \hbar \beta k \Lambda \quad (9)$$

where

$$\begin{aligned} \Gamma \equiv & \left\{ \frac{1}{3} \left[ \frac{1}{\{\alpha^2 + (k-p)^2\}^3} - \frac{1}{\{\alpha^2 + (k+p)^2\}^3} \right] \right. \\ & - \frac{32\alpha^4}{(\alpha^2 + p^2)^2 (4\alpha^2 + k^2)^2} \left[ \frac{1}{\alpha^2 + (k-p)^2} - \frac{1}{\alpha^2 + (k+p)^2} \right] \\ & \left. + \frac{1024pk\alpha^8}{(\alpha^2 + p^2)^4 (4\alpha^2 + k^2)^4} \right\} , \end{aligned} \quad (10)$$

$$\Lambda \equiv \left[ \omega_B + \frac{\hbar\beta}{m} (k^2 + \alpha^2) \right] \left[ \frac{1}{8\alpha^5} - \frac{32\alpha^3}{(4\alpha^2 + k^2)^4} \right] , \quad (11)$$

and

$$p = [m(\omega - \omega_B)/\hbar\beta]^{1/2} . \quad (12)$$

Here  $\hbar\omega_B$  is the average binding energy of the valence band and  $\beta$  and  $\alpha$  are parameters which may be adjusted to make the theory agree with optical dielectric function measurements in the  $k \rightarrow 0$  limit. In the  $k \rightarrow 0$  limit we have

$$\text{Im}[\epsilon(0, \omega)] = \frac{2^9 \pi n e^2}{3 \hbar \beta} \frac{\alpha^7}{(\omega_B + \frac{\hbar \beta}{m} \alpha^2)} \frac{p^3}{(\alpha^2 + p^2)^6} \quad (13)$$

Given the imaginary part of the dielectric function, Eq. (9), for fixed values of  $n$ ,  $\beta$ ,  $\omega_B$ , and  $\alpha$  the real part of  $\epsilon(k, \omega)$  may be obtained numerically using the Kramers-Kronig relation,

$$\text{Re}[\epsilon(k, \omega)] = 1 + \frac{2P}{\pi} \int_0^\infty d\omega_o \frac{\omega_o \text{Im}[\epsilon(k, \omega_o)]}{\omega_o^2 - \omega^2} \quad (14)$$

Equation (13) is used to fit experimental data on  $\text{Im}[\epsilon(0, \omega)]$ , as obtained from optical measurements or from electron energy loss measurements, to determine parameters for the insulator model. Since the general shape of this function for most semiconductors shows two major peaks separated by a few eV, two terms of the form on the right-hand side of Eq. (13) (corresponding to the combination of two single orbitals in the OPW excited state wave function) were used to provide a reasonable fit to the data.<sup>4</sup> More specifically, the parameters for the tabulations were determined in the following manner:

#### A. Germanium

For 4 electrons per atom contributing to the valence band, and for a density of  $5.30 \text{ gm/cm}^3$  for the solid Ge



an initial set of parameters was determined by fitting the optical data of Ref. 16 to obtain  $\text{Im}[\epsilon(o,\omega)]$ . From Eq. (14)  $\text{Re}[\epsilon(o,\omega)]$  is determined and  $\text{Im}[-1/\epsilon(o,\omega)]$  is computed for comparison with the values obtained for this function using energy-loss spectroscopy.<sup>17</sup> Small changes were made in the initial set of parameters so that a good fit was obtained to the measured  $\text{Im}[-1/\epsilon(o,\omega)]$ . The parameter values obtained in this manner are:

- (a) (.9) of the 4 valence electrons:  $\hbar\omega_{B_1} = 1.9 \text{ eV}$ ,  
 $\alpha_1 a_0 = 0.485$ ,  $\beta_1 = 0.5$
- (b) (.1) of the 4 valence electrons:  $\hbar\omega_{B_2} = 3.9 \text{ eV}$ ,  
 $\alpha_2 a_0 = 0.19$ ,  $\beta_2 = 0.5$ .

Given these values,  $\text{Im}[-1/\epsilon(k,\omega)]$  is calculated using Eq. (9) and Eq. (14) to determine the DIMFP for interaction with the valence band electrons. The contribution of this valence band resulting from the combination of these two orbitals will be identified in the tables by VALENCE.

#### B. Gallium Arsenide

For application of the model insulator theory to GaAs we used experimental data obtained entirely from electron energy loss spectroscopy.<sup>17</sup> As with Ge, an initial set of parameters was obtained from data on  $\text{Im}[\epsilon(o,\omega)]$  and the final set of values obtained by requiring a good fit to  $\text{Im}[-1/\epsilon(o,\omega)]$ . With 8 valence

electrons per molecule and a density of 5.31 gm/cm<sup>3</sup> for solid GaAs, the parameters were determined to be:

(a) (.95) of the 8 valence electrons:  $\hbar\omega_{B_1} = 2.45$  eV,

$$\alpha_1 a_0 = 0.53, \beta_1 = 0.5$$

(b) (.05) of the 8 valence electrons:  $\hbar\omega_{B_2} = 4.3$  eV,

$$\alpha_2 a_0 = 0.34, \beta_2 = 0.5.$$

Given these values,  $\text{Im} [-1/\epsilon(k, \omega)]$  is calculated using Eq. (9) and Eq. (14) to determine the DIMFP for interaction with valence band electrons. The contribution of this valence band will be identified in the tables by VALENCE.

#### IV. DIMFP's FOR INNER SHELLS

From a general expression for the dielectric function of a homogeneous, isotropic system<sup>18</sup> we may show for values of  $\omega$  which correspond to ionization of the  $i^{\text{th}}$  inner shell in a solid that

$$\text{Im} \left[ \frac{-1}{\epsilon(k, \omega)} \right] \approx \text{Im} \epsilon(k, \omega) \approx \frac{2\pi n_i e^2}{m\omega} \frac{df_i(k, \omega)}{d\omega} \quad (15)$$

where  $df_i/d\omega$  is the generalized oscillator strength (GOS) for transitions from the  $i^{\text{th}}$  level in the atom to a continuum final state. Here  $n_i$  is the number of electrons in the  $i^{\text{th}}$  level per unit volume in the given solid. Equation (2) thus leads to

$$\tau_i(E, \hbar\omega) = \frac{2\pi e^4 n_i}{E \hbar\omega} \int_{k_-}^{k_+} \frac{dk}{k} \frac{df_i(k, \omega)}{d(\hbar\omega)} \quad (16)$$

where  $\hbar\omega$  is the energy transfer. In terms of the doubly-differential cross section per electron for excitation into continuum final states this equation becomes

$$\tau_i(E, \hbar\omega) = n_i \int_{k_-}^{k_+} dk \frac{d^2\sigma_i}{dkd(\hbar\omega)}, \quad (17)$$

where

$$\frac{d^2\sigma_i}{dkd(\hbar\omega)} = \frac{2\pi e^4}{E} \frac{1}{k} \frac{1}{\hbar\omega} \frac{df_i(k, \omega)}{d(\hbar\omega)}. \quad (18)$$

The basic doubly-differential cross section in the classical binary collision (CBC) model is given for spherically symmetric distributions of velocities of the atomic electrons by<sup>6,7</sup>

$$\frac{d^2\sigma_i}{dkd(\hbar\omega)} = \frac{2\pi m e^4}{\hbar^3 E v_i k^4} \theta(\hbar\omega - \epsilon_i) \theta(E - \hbar\omega) \quad (19)$$

where

$E$  = energy of the incident electron,

$v_i$  = speed of struck electron in the  $i^{\text{th}}$  level =  $\sqrt{2E_i/m}$ ,

$\epsilon_i$  = "ionization energy" of struck electron,

$\theta(x) = (0 \text{ for } x < 0; 1 \text{ for } x > 0)$ .

From conservation of energy and momentum the limits on the integration in Eq. (17) for the CBC model are given by

$$\hbar k_{\pm} = \sqrt{2m} \begin{cases} \sqrt{E_i + \hbar\omega} \pm \sqrt{E_i} & , \text{ for } E \geq \hbar\omega + E_i \\ \sqrt{E} \pm \sqrt{E - \hbar\omega} & , \text{ for } E < \hbar\omega + E_i. \end{cases} \quad (20)$$



For a given atomic level the function  $\frac{d^2 \sigma_i}{dk d(\hbar\omega)}$ , Eq. (19), becomes a function also of  $v_i$ , and is averaged over a distribution of electron speeds for this level. We use here the hydrogenic speed distribution given by<sup>8,9</sup>

$$f(v_i) = \left(\frac{32}{\pi}\right) v_i^2 v_{oi}^5 (v_i^2 + v_{oi}^2)^{-4} \quad (21)$$

where  $v_{oi}$  is the mean speed of electrons in the  $i^{\text{th}}$  level and is set equal to  $\sqrt{2\epsilon_i/m}$ . The GOS, Eq. (18), and DIMFP, Eq. (17), averaged over this speed distribution for the  $i^{\text{th}}$  level are defined by

$$\frac{df_i^{\text{AV}}(k, \omega)}{d(\hbar\omega)} = \int_0^\infty dv_i f(v_i) \frac{df_i(k, \omega)}{d(\hbar\omega)}, \quad (22)$$

$$\tau_i^{\text{AV}}(E, \hbar\omega) = \int_0^\infty dv_i f(v_i) \tau_i(E, \hbar\omega). \quad (23)$$

The results of these integrations are<sup>8,9</sup>

$$\frac{df_i^{\text{AV}}(n, x)}{dx} = \frac{2^8}{3\pi} \frac{x n^3}{[(x-n)^2 + 4n^2]^3} \quad (24)$$

$$\tau_i^{\text{AV}}(\beta, x) = \frac{A_i}{3\pi a_0} \frac{\theta(x-1)}{\beta x^3} \left[ (3x+4) \left( \tan^{-1} y + \frac{y}{1+y^2} \right) + \frac{2y(x-4)}{(1+y^2)^2} \right], \quad (25)$$

where the variables are defined by

$$\eta = \hbar k / \sqrt{2m\epsilon_i} ,$$

$$\beta = E/\epsilon_i ,$$

$$x = \hbar\omega / \epsilon_i ,$$

$$y = \sqrt{\beta - x} ,$$

$$A_i = 8\pi a_0^3 n_i (R/\epsilon_i)^2 ,$$

$$a_0 = \hbar^2 / me^2 = 0.529 \text{ \AA} ,$$

$$R = e^2 / 2a_0 = 13.6 \text{ eV} .$$

It is well known that the CBC cross sections do not satisfy the fundamental Bethe sum rule<sup>9</sup>. It is straightforward to show that if

$$f_i^{AV}(\eta) = \int_1^\infty \frac{df_i^{AV}(\eta, x) dx}{x} \quad (26)$$

then from Eq. (24),  $f_i^{AV}(\eta) \rightarrow 1$  as  $\eta \rightarrow \infty$  as it should, but that  $f_i^{AV}(\eta) \rightarrow 0$  as  $\eta \rightarrow 0$ . To remedy this deficiency in an approximate way we may add oscillator strength at energy losses around  $\epsilon_i$  with  $k$ -dependent amplitude  $1 - f_i^{AV}(k)$ . We approximate the true GOS of a given shell by

$$\frac{df_i(k, \omega)}{d\omega} = \frac{df_i(o, \omega)}{d\omega} [1 - f_i^{AV}(k)] + \frac{df_i^{AV}(k, \omega)}{d\omega} , \quad (27)$$

where  $\frac{df_i(o, \omega)}{d\omega}$  is the known optical oscillator strength. That is, we have used an interpolation scheme for the GOS which reduces to the optical oscillator strength for small momentum transfers and which goes over into the CBC GOS values for large momentum transfer.

We have used a form

$$\frac{df_i(o, \omega)}{d\omega} = C \frac{(\omega - \epsilon_i)}{(a + \omega)^{\alpha+1}} \quad (28)$$

to represent the optical oscillator strength of a given shell and adjusted parameters  $a$  and  $\alpha$  so that the total stopping power including that of the valence band agrees with the Bethe formula in the high energy region. Parameters chosen for inner shells of both Ge and GaAs are  $\alpha = 3.5$  and  $a = 0.165$ .

From the  $\tau_i$ , obtained from Eq. (17), (18), and (21), we calculate an exchange corrected DIMFP from Eq. (35) of Section V. Then using Eqs. (36)-(38) of Section V, we calculate exchange corrected inverse mean free path, stopping power, and mean square energy loss which may be written in the form

$$\mu_i(\beta) = \frac{4n_i a_o^2}{(\epsilon_i/R)^2} F(\beta), \quad (29)$$

$$S_i(\beta) = \frac{4\epsilon_i n_i a_o^2}{(\epsilon_i/R)^2} G(\beta), \quad (30)$$

$$\Omega_i^2(\beta) = \frac{4\epsilon_i^2 n_i a_o^2}{(\epsilon_i/R)^2} H(\beta). \quad (31)$$

The universal functions  $F$ ,  $G$ , and  $H$  defined by these equations are tabulated in Table 3 as a function of  $\beta$ .



## V. EXCHANGE CORRECTED DIMFP'S AND FORMULAE FOR THE TABULATIONS

We have included the effect of electron exchange in our calculations in a simple manner based on the form of the Mott formula (nonrelativistic Møller formula) for scattering of an incident electron with a free electron. The cross section for finding a scattered electron with energy  $W$  per unit energy interval is given by<sup>11</sup>

$$\frac{d\phi}{dW} = \frac{\pi e^4}{E} \left[ \frac{1}{W^2} + \frac{1}{(E-W)^2} - \frac{1}{W(E-W)} \right] \quad (32)$$

for an incident electron of energy  $E$ , except for energies close to  $W = 0$  and  $W = E$ . Near  $W = 0$  and  $W = E$  the interference term (third term on the right side of Eq. (32)) is effectively zero.

The DIMFP for excitation of an electron from a particular state  $i$  may be written in the form

$$\tau_i(E, \hbar\omega) = \frac{1}{E} F_i(E, \hbar\omega). \quad (33)$$

If we assume that the width of the level from which an electron is excited is quite narrow, we obtain from Eq. (33) the DIMFP for production of a secondary electron with energy  $E_s$  as

$$\tau_i^s(E, E_s) = \frac{1}{E} F_i(E, E_i^B + E_s) \quad (34)$$

where  $E_i^B$  is the binding energy of the  $i^{\text{th}}$  level (a positive quantity).

The exchange corrected DIMFP is taken as

$$\tau_i^{\text{exc}}(E, \hbar\omega) = \frac{1}{E} \left\{ F_i(E, \hbar\omega) + F_i(E, E+E_i^B-\hbar\omega) - \left[ 1 - \sqrt{E_i^B/E} \right] \left[ F_i(E, \hbar\omega) F_i(E, E+E_i^B-\hbar\omega) \right]^{\frac{1}{2}} \right\} \quad (35)$$

Since  $E\tau_i \propto 1/(\hbar\omega)^2$  for large  $E$  and  $\hbar\omega$ , Eq. (35) reduces in this limit to the form given by Eq. (32). The factor  $1 - \sqrt{E_i^B/E}$  reduces the contribution of the third term in Eq. (35) as  $E \rightarrow E_i^B$ . This form for the exchange corrected DIMFP has been used in our calculations for all the inner shells and for the valence bands (since our model assumes the width of these levels to be quite narrow).

If we now define the more energetic of the two electrons after collision to be the primary and account for exchange through Eq. (35), Eq. (3) gives the contribution to the inverse mean free path due to excitation of an electron from the  $i^{\text{th}}$  level as

$$\mu_i(E) = \int_{E_i^B}^{(E+E_i^B)/2} d(\hbar\omega) \tau_i^{\text{exc}}(E, \hbar\omega) \quad (36)$$

Similarly, for the stopping power and mean square energy loss per unit path length, we have from Eq. (4) and Eq. (5)

$$S_i(E) = \int_{E_i^B}^{(E+E_i^B)/2} d(\hbar\omega) \hbar\omega \tau_i^{\text{exc}}(E, \hbar\omega) \quad (37)$$

and

$$\Omega_i^2(E) = \int_{E_i^B}^{(E+E_i^B)/2} d(\hbar\omega) (\hbar\omega)^2 \tau_i^{\text{exc}}(E, \hbar\omega). \quad (38)$$

For the remaining calculations we form the sums

$$S_{\text{exc}}(E) = \sum_i S_i(E) \quad (39)$$

and

$$\Omega_{\text{exc}}^2(E) = \sum_i \Omega_i^2(E) \quad (40)$$

where the index  $i$  includes the terms appropriate for a given solid, including exchange corrections as indicated above. The csda range is calculated from

$$R_{(10)}(E) = \int_{10\text{eV}}^E dE' / S_{\text{exc}}(E') \quad (41)$$

corresponding to an electron slowing down in a continuous manner from an energy  $E$  to 10 eV. The mean square fluctuation in the csda range based on Eq. (7) is calculated as

$$[\Delta R_{(10)}]_{\text{AV}}^2 = \int_{10\text{ eV}}^E dE' \Omega_{\text{exc}}^2(E') / [S_{\text{exc}}(E')]^3. \quad (42)$$



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## VII. GERMANIUM: EXPLANATION OF TABLES

### General Notes

1. Electron energies are measured from the bottom of the conduction band.
2. The density of solid Ge is taken to be  $5.30\text{g/cm}^3$ .
3. The computer-printed units are translated as:  

EV.....eV	A..... $\text{\AA}$
EV2.....(eV) <sup>2</sup>	A-1..... $\text{\AA}^{-1}$
G/CM3.....g/cm <sup>3</sup>	A2..... $\text{\AA}^2$
4. The numerical printout is in the form, e.g.,  
 $2.8\text{D}-01 \equiv 2.8 \times 10^{-1}$

TABLE 1A — INVERSE MEAN FREE PATH OF ELECTRONS IN GERMANIUM

INNER SH.	Inner shell contribution to inverse mean free path
VALENCE	Valence band contribution to inverse mean free path
TOTAL	$\mu$ - total inverse mean free path = sum of inner shell and valence band contributions

TABLE 1B — STOPPING POWER OF GERMANIUM FOR ELECTRONS

INNER SH.	Inner shell contribution to stopping power
VALENCE	Valence band contribution to stopping power
TOTAL	S - total stopping power = sum of inner shell and valence band contributions



TABLE 1C — CSDA RANGE AND STRAGGLING OF ELECTRONS IN GERMANIUM

CSDA RANGE (E - 10 EV)	$R_{(10)}$ - the range of an electron in the continuous-slowing-down approximation in going from an energy E to 10 eV.
MEAN SQ. EN. LOSS	$\Omega_{exc}^2$ - the mean square fluctuation in the energy loss per unit path length
MEAN SQ. RANGE FL.	$[\Delta R_{(10)}]^2_{AV}$ - the mean square fluctuation in the range about the mean csda range $R_{(10)}$
RELATIVE RANGE STRAGGLING	$\{[\Delta R_{(10)}]^2_{AV}\}^{1/2}/R_{(10)}$

TABLE 1A-INVERSE MEAN FREE PATH OF ELECTRONS IN GERMANIUM

ELECTRON ENERGY EV	INVERSE MFP IN UNITS OF A-1		
	INNER SH.	VALENCE	TOTAL
1.000 00	0.0	0.0	0.0
1.500 00	0.0	0.0	0.0
2.000 00	0.0	1.8840-03	1.8840-03
2.500 00	0.0	2.7130-03	2.7130-03
3.000 00	0.0	3.7040-03	3.7040-03
3.500 00	0.0	4.8410-03	4.8410-03
4.000 00	0.0	6.0310-03	6.0310-03
4.500 00	0.0	7.3120-03	7.3120-03
5.000 00	0.0	8.7860-03	8.7860-03
5.500 00	0.0	1.0210-02	1.0210-02
6.000 00	0.0	1.1830-02	1.1830-02
6.500 00	0.0	1.3550-02	1.3550-02
7.000 00	0.0	1.5280-02	1.5280-02
7.500 00	0.0	1.7320-02	1.7320-02
8.000 00	0.0	1.9910-02	1.9910-02
8.500 00	0.0	2.2290-02	2.2290-02
9.000 00	0.0	2.4360-02	2.4360-02
9.500 00	0.0	2.6420-02	2.6420-02
1.000 01	0.0	2.8540-02	2.8540-02
1.500 01	0.0	5.3600-02	5.3600-02
2.000 01	0.0	8.5520-02	8.5520-02
2.500 01	0.0	1.2690-01	1.2690-01
3.000 01	0.0	1.6010-01	1.6010-01
3.500 01	0.0	1.7980-01	1.7980-01
4.000 01	4.1150-03	1.9230-01	1.9640-01
4.500 01	9.9650-03	2.0280-01	2.1280-01
5.000 01	1.8520-02	2.0980-01	2.2830-01
5.500 01	2.8200-02	2.1340-01	2.4160-01
6.000 01	3.5490-02	2.1430-01	2.4970-01
6.500 01	4.2010-02	2.1360-01	2.5560-01
7.000 01	4.6980-02	2.1150-01	2.5850-01
7.500 01	5.0900-02	2.0860-01	2.5950-01
8.000 01	5.3950-02	2.0540-01	2.5930-01
8.500 01	5.6120-02	2.0160-01	2.5770-01
9.000 01	5.7840-02	1.9770-01	2.5550-01
9.500 01	5.9160-02	1.9380-01	2.5290-01
1.000 02	6.0040-02	1.8990-01	2.4990-01
1.500 02	5.9560-02	1.5510-01	2.1460-01
2.000 02	5.5440-02	1.3030-01	1.8570-01
2.500 02	5.1030-02	1.1250-01	1.6350-01
3.000 02	4.6970-02	9.9150-02	1.4610-01
3.500 02	4.3400-02	8.8830-02	1.3220-01
4.000 02	4.0300-02	8.0590-02	1.2090-01
4.500 02	3.7600-02	7.3830-02	1.1140-01
5.000 02	3.5250-02	6.8200-02	1.0350-01
5.500 02	3.3190-02	6.3420-02	9.6610-02
6.000 02	3.1360-02	5.9300-02	9.0670-02
6.500 02	2.9740-02	5.5730-02	8.5470-02
7.000 02	2.8280-02	5.2600-02	8.0880-02
7.500 02	2.6980-02	4.9820-02	7.6800-02
8.000 02	2.5790-02	4.7350-02	7.3140-02
8.500 02	2.4710-02	4.5120-02	6.9840-02
9.000 02	2.3730-02	4.3110-02	6.6840-02
9.500 02	2.2830-02	4.1280-02	6.4110-02
1.000 03	2.1990-02	3.9610-02	6.1610-02
1.500 03	1.6260-02	2.8480-02	4.4740-02
2.000 03	1.3040-02	2.2440-02	3.5470-02
2.500 03	1.0940-02	1.8610-02	2.9550-02
3.000 03	9.4580-03	1.5960-02	2.5410-02
3.500 03	8.3510-03	1.4000-02	2.2350-02
4.000 03	7.4900-03	1.2490-02	1.9980-02
4.500 03	6.8000-03	1.1290-02	1.8090-02
5.000 03	6.2330-03	1.0320-02	1.6550-02
5.500 03	5.7590-03	9.5050-03	1.5260-02
6.000 03	5.3560-03	8.8170-03	1.4170-02
6.500 03	5.0090-03	8.2280-03	1.3240-02
7.000 03	4.7070-03	7.7160-03	1.2420-02
7.500 03	4.4410-03	7.2580-03	1.1710-02
8.000 03	4.2060-03	6.8720-03	1.1080-02
8.500 03	3.9950-03	6.5190-03	1.0510-02
9.000 03	3.8060-03	6.2020-03	1.0010-02
9.500 03	3.6350-03	5.9170-03	9.5520-03
1.000 04	3.4800-03	5.6580-03	9.1380-03

TABLE 1B-STOPPING POWER OF GERMANIUM FOR ELECTRONS

ELECTRON ENERGY EV	STOPPING POWER IN UNITS OF EV/A		
	INNER SH.	VALENCE	TOTAL
1.000 00	0.0	0.0	0.0
1.500 00	0.0	0.0	0.0
2.000 00	0.0	3.761D-03	3.761D-03
2.500 00	0.0	5.415D-03	5.415D-03
3.000 00	0.0	8.401D-03	8.401D-03
3.500 00	0.0	1.141D-02	1.141D-02
4.000 00	0.0	1.563D-02	1.563D-02
4.500 00	0.0	2.026D-02	2.026D-02
5.000 00	0.0	2.617D-02	2.617D-02
5.500 00	0.0	3.225D-02	3.225D-02
6.000 00	0.0	3.943D-02	3.943D-02
6.500 00	0.0	4.733D-02	4.733D-02
7.000 00	0.0	5.651D-02	5.651D-02
7.500 00	0.0	6.641D-02	6.641D-02
8.000 00	0.0	7.810D-02	7.810D-02
8.500 00	0.0	9.111D-02	9.111D-02
9.000 00	0.0	1.060D-01	1.060D-01
9.500 00	0.0	1.174D-01	1.174D-01
1.000 01	0.0	1.298D-01	1.298D-01
1.500 01	0.0	3.090D-01	3.090D-01
2.000 01	0.0	6.230D-01	6.230D-01
2.500 01	0.0	1.150D 00	1.150D 00
3.000 01	0.0	1.796D 00	1.796D 00
3.500 01	0.0	2.398D 00	2.398D 00
4.000 01	1.623D-01	2.901D 00	3.063D 00
4.500 01	4.060D-01	3.295D 00	3.701D 00
5.000 01	7.773D-01	3.593D 00	4.370D 00
5.500 01	1.217D 00	3.799D 00	5.016D 00
6.000 01	1.572D 00	3.912D 00	5.484D 00
6.500 01	1.905D 00	3.982D 00	5.888D 00
7.000 01	2.179D 00	4.011D 00	6.189D 00
7.500 01	2.410D 00	4.009D 00	6.419D 00
8.000 01	2.604D 00	3.993D 00	6.598D 00
8.500 01	2.758D 00	3.960D 00	6.718D 00
9.000 01	2.891D 00	3.918D 00	6.809D 00
9.500 01	3.004D 00	3.871D 00	6.875D 00
1.000 02	3.094D 00	3.822D 00	6.916D 00
1.500 02	3.483D 00	3.271D 00	6.754D 00
2.000 02	3.614D 00	2.821D 00	6.435D 00
2.500 02	3.611D 00	2.480D 00	6.091D 00
3.000 02	3.532D 00	2.216D 00	5.748D 00
3.500 02	3.421D 00	2.008D 00	5.428D 00
4.000 02	3.297D 00	1.838D 00	5.136D 00
4.500 02	3.173D 00	1.697D 00	4.871D 00
5.000 02	3.053D 00	1.579D 00	4.632D 00
5.500 02	2.939D 00	1.477D 00	4.416D 00
6.000 02	2.831D 00	1.389D 00	4.220D 00
6.500 02	2.731D 00	1.311D 00	4.042D 00
7.000 02	2.637D 00	1.243D 00	3.880D 00
7.500 02	2.550D 00	1.182D 00	3.732D 00
8.000 02	2.468D 00	1.128D 00	3.596D 00
8.500 02	2.392D 00	1.078D 00	3.470D 00
9.000 02	2.321D 00	1.034D 00	3.354D 00
9.500 02	2.254D 00	9.926D-01	3.247D 00
1.000 03	2.191D 00	9.551D-01	3.147D 00
1.500 03	1.741D 00	7.010D-01	2.442D 00
2.000 03	1.483D 00	5.598D-01	2.042D 00
2.500 03	1.304D 00	4.689D-01	1.773D 00
3.000 03	1.170D 00	4.051D-01	1.575D 00
3.500 03	1.064D 00	3.576D-01	1.421D 00
4.000 03	9.775D-01	3.208D-01	1.298D 00
4.500 03	9.059D-01	2.913D-01	1.197D 00
5.000 03	8.452D-01	2.671D-01	1.112D 00
5.500 03	7.930D-01	2.469D-01	1.040D 00
6.000 03	7.476D-01	2.298D-01	9.773D-01
6.500 03	7.077D-01	2.150D-01	9.227D-01
7.000 03	6.723D-01	2.021D-01	8.744D-01
7.500 03	6.407D-01	1.908D-01	8.315D-01
8.000 03	6.122D-01	1.808D-01	7.930D-01
8.500 03	5.865D-01	1.718D-01	7.583D-01
9.000 03	5.630D-01	1.638D-01	7.268D-01
9.500 03	5.416D-01	1.565D-01	6.981D-01
1.000 04	5.219D-01	1.499D-01	6.718D-01



TABLE 1C-CSDA RANGE AND STRAGGLING OF ELECTRONS IN GERMANIUM

ELECTRON ENERGY EV	CSDA RANGE (E - 10EV) A	MEAN SQ. EN. LOSS EV/A	MEAN SQ. RANGE FL. A2	RELATIVE RANGE STRAGGLING
1.500 01	2.5320 01	6.1210-01	7.1590 02	1.0570 00
2.000 01	3.6860 01	1.9310 00	9.0740 02	8.1730-01
2.500 01	4.2750 01	4.9860 00	9.7020 02	7.2860-01
3.000 01	4.6200 01	1.1450 01	9.9670 02	6.8330-01
3.500 01	4.8610 01	2.1990 01	1.0120 03	6.5440-01
4.000 01	5.0450 01	3.4740 01	1.0230 03	6.3400-01
4.500 01	5.1930 01	5.3850 01	1.0310 03	6.1830-01
5.000 01	5.3180 01	7.4570 01	1.0380 03	6.0580-01
5.500 01	5.4240 01	9.9540 01	1.0430 03	5.9550-01
6.000 01	5.5190 01	1.2650 02	1.0480 03	5.8660-01
6.500 01	5.6070 01	1.4840 02	1.0520 03	5.7860-01
7.000 01	5.6900 01	1.6910 02	1.0560 03	5.7120-01
7.500 01	5.7690 01	1.8670 02	1.0600 03	5.6440-01
8.000 01	5.8460 01	2.0210 02	1.0640 03	5.5800-01
8.500 01	5.9210 01	2.1560 02	1.0680 03	5.5190-01
9.000 01	5.9950 01	2.2690 02	1.0720 03	5.4600-01
9.500 01	6.0680 01	2.3700 02	1.0750 03	5.4040-01
1.000 02	6.1400 01	2.4610 02	1.0790 03	5.3500-01
1.500 02	6.8650 01	2.5400 02	1.1220 03	4.8800-01
2.000 02	7.6230 01	3.0840 02	1.1810 03	4.5070-01
2.500 02	8.4210 01	3.5670 02	1.2570 03	4.2100-01
3.000 02	9.2660 01	3.9270 02	1.3550 03	3.9730-01
3.500 02	1.0160 02	4.1620 02	1.4770 03	3.7820-01
4.000 02	1.1110 02	4.3140 02	1.6260 03	3.6300-01
4.500 02	1.2110 02	4.4030 02	1.8040 03	3.5070-01
5.000 02	1.3160 02	4.4660 02	2.0130 03	3.4090-01
5.500 02	1.4270 02	4.4990 02	2.2570 03	3.3300-01
6.000 02	1.5430 02	4.5130 02	2.5380 03	3.2660-01
6.500 02	1.6640 02	4.5170 02	2.8590 03	3.2140-01
7.000 02	1.7900 02	4.5120 02	3.2220 03	3.1710-01
7.500 02	1.9210 02	4.5010 02	3.6300 03	3.1360-01
8.000 02	2.0580 02	4.4870 02	4.0860 03	3.1060-01
8.500 02	2.2000 02	4.4690 02	4.5920 03	3.0810-01
9.000 02	2.3460 02	4.4500 02	5.1520 03	3.0590-01
9.500 02	2.4980 02	4.4290 02	5.7670 03	3.0400-01
1.000 03	2.6540 02	4.4080 02	6.4410 03	3.0240-01
1.500 03	4.4770 02	4.3860 02	1.7140 04	2.9240-01
2.000 03	6.7280 02	4.3570 02	3.7820 04	2.8900-01
2.500 03	9.3650 02	4.6350 02	7.2860 04	2.8820-01
3.000 03	1.2360 03	4.8570 02	1.2630 05	2.8750-01
3.500 03	1.5710 03	4.9990 02	2.0220 05	2.8620-01
4.000 03	1.9400 03	5.0860 02	3.0480 05	2.8460-01
4.500 03	2.3410 03	5.1360 02	4.3830 05	2.8280-01
5.000 03	2.7750 03	5.1610 02	6.0710 05	2.8080-01
5.500 03	3.2400 03	5.1720 02	8.1560 05	2.7870-01
6.000 03	3.7360 03	5.1700 02	1.0680 06	2.7670-01
6.500 03	4.2630 03	5.1630 02	1.3700 06	2.7460-01
7.000 03	4.8200 03	5.1500 02	1.7260 06	2.7260-01
7.500 03	5.4060 03	5.1350 02	2.1400 06	2.7060-01
8.000 03	6.0220 03	5.1170 02	2.6180 06	2.6860-01
8.500 03	6.6670 03	5.0980 02	3.1640 06	2.6680-01
9.000 03	7.3410 03	5.0780 02	3.7840 06	2.6500-01
9.500 03	8.0430 03	5.0570 02	4.4830 06	2.6320-01
1.000 04	8.7740 03	5.0370 02	5.2660 06	2.6160-01

# VIII. GALLIUM ARSENIDE: EXPLANATION OF TABLES

## General Notes

1. Electron energies are measured from the bottom of the conduction band.
2. The density of solid GaAs is taken to be  $5.31 \text{ g/cm}^3$ .
3. The computer-printed units are translated as:
 

EV.....eV	A..... $\text{\AA}$
EV2.....(eV) <sup>2</sup>	A-1..... $\text{\AA}^{-1}$
G/CM3.....g/cm <sup>3</sup>	A2..... $\text{\AA}^2$
4. The numerical printout is in the form, e.g.,  $2.8\text{D}-2 \equiv 2.8 \times 10^{-2}$ .

TABLE 2A — INVERSE MEAN FREE PATH OF ELECTRON IN GALLIUM ARSENIDE

INNER SH.	Inner shell contribution to inverse mean free path
VALENCE	Valence band contribution to inverse mean free path
TOTAL	$\mu$ - total inverse mean free path = sum of inner shell and valence band contributions

TABLE 2B — STOPPING POWER OF GALLIUM ARSENIDE FOR ELECTRONS

INNER SH.	Inner shell contribution to stopping power
VALENCE	Valence band contribution to stopping power
TOTAL	S - total stopping power = sum of inner shell and valence band contributions

TABLE 2C — CSDA RANGE AND STRAGGLING OF ELECTRONS IN GALLIUM ARSENIDE

CSDA RANGE (E-10EV)	$R_{(10)}$ - the range of an electron in the continuous-slowing-down approximation in going from an energy E to 10 eV
MEAN SQ. EN. LOSS	$\Omega_{exc}^2$ - the mean square fluctuation in the energy loss per unit path length
MEAN SQ. RANGE FL.	$[\Delta R_{(10)}]_{AV}^2$ - the mean square fluctuation in the range about the mean csda range $R_{(10)}$
RELATIVE RANGE STRAGGLING	$\{[\Delta R_{(10)}]_{AV}^2\}^{1/2}/R_{(10)}$



TABLE 2A-INVERSE MEAN FREE PATH OF ELECTRONS IN GALLIUM ARSENIDE

ELECTRON ENERGY eV	INVERSE MFP IN UNITS OF A-1		
	INNER SH.	VALENCE	TOTAL
1.000 00	0.0	0.0	0.0
1.500 00	0.0	0.0	0.0
2.000 00	0.0	3.1070-03	3.1070-03
2.500 00	0.0	4.3620-03	4.3620-03
3.000 00	0.0	5.7410-03	5.7410-03
3.500 00	0.0	7.2290-03	7.2290-03
4.000 00	0.0	8.8150-03	8.8150-03
4.500 00	0.0	1.0490-02	1.0490-02
5.000 00	0.0	1.2240-02	1.2240-02
5.500 00	0.0	1.4070-02	1.4070-02
6.000 00	0.0	1.5970-02	1.5970-02
6.500 00	0.0	1.7940-02	1.7940-02
7.000 00	0.0	1.9960-02	1.9960-02
7.500 00	0.0	2.2040-02	2.2040-02
8.000 00	0.0	2.4180-02	2.4180-02
8.500 00	0.0	2.6370-02	2.6370-02
9.000 00	0.0	2.8610-02	2.8610-02
9.500 00	0.0	3.0890-02	3.0890-02
1.000 01	0.0	3.3230-02	3.3230-02
1.500 01	0.0	5.8660-02	5.8660-02
2.000 01	0.0	8.7180-02	8.7180-02
2.500 01	0.0	1.1930-01	1.1930-01
3.000 01	6.7220-03	1.4700-01	1.5370-01
3.500 01	1.7910-02	1.6620-01	1.8410-01
4.000 01	3.1000-02	1.7930-01	2.1030-01
4.500 01	4.0880-02	1.8920-01	2.3010-01
5.000 01	4.8150-02	1.9580-01	2.4400-01
5.500 01	5.4990-02	1.9950-01	2.5450-01
6.000 01	5.9690-02	2.0070-01	2.6040-01
6.500 01	6.3870-02	2.0050-01	2.6430-01
7.000 01	6.7470-02	1.9900-01	2.6640-01
7.500 01	7.0080-02	1.9660-01	2.6670-01
8.000 01	7.1970-02	1.9380-01	2.6580-01
8.500 01	7.3360-02	1.9050-01	2.6390-01
9.000 01	7.4250-02	1.8710-01	2.6130-01
9.500 01	7.4720-02	1.8350-01	2.5830-01
1.000 02	7.4930-02	1.8000-01	2.5500-01
1.500 02	7.0840-02	1.4790-01	2.1870-01
2.000 02	6.4520-02	1.2460-01	1.8910-01
2.500 02	5.8680-02	1.0770-01	1.6640-01
3.000 02	5.3620-02	9.4980-02	1.4860-01
3.500 02	4.9300-02	8.5120-02	1.3440-01
4.000 02	4.5610-02	7.7240-02	1.2290-01
4.500 02	4.2440-02	7.0760-02	1.1320-01
5.000 02	3.9700-02	6.5360-02	1.0510-01
5.500 02	3.7300-02	6.0780-02	9.8090-02
6.000 02	3.5190-02	5.6840-02	9.2030-02
6.500 02	3.3330-02	5.3410-02	8.6740-02
7.000 02	3.1660-02	5.0400-02	8.2060-02
7.500 02	3.0170-02	4.7740-02	7.7910-02
8.000 02	2.8820-02	4.5370-02	7.4180-02
8.500 02	2.7590-02	4.3230-02	7.0820-02
9.000 02	2.6470-02	4.1290-02	6.7760-02
9.500 02	2.5450-02	3.9540-02	6.4980-02
1.000 03	2.4500-02	3.7940-02	6.2440-02
1.500 03	1.8050-02	2.7250-02	4.5300-02
2.000 03	1.4430-02	2.1460-02	3.5890-02
2.500 03	1.2090-02	1.7800-02	2.9880-02
3.000 03	1.0440-02	1.5250-02	2.5690-02
3.500 03	9.2090-03	1.3380-02	2.2590-02
4.000 03	8.2540-03	1.1940-02	2.0190-02
4.500 03	7.4890-03	1.0790-02	1.8280-02
5.000 03	6.8610-03	9.8560-03	1.6720-02
5.500 03	6.3370-03	9.0790-03	1.5420-02
6.000 03	5.8910-03	8.4210-03	1.4310-02
6.500 03	5.5080-03	7.8570-03	1.3370-02
7.000 03	5.1740-03	7.3630-03	1.2540-02
7.500 03	4.8810-03	6.9400-03	1.1820-02
8.000 03	4.6210-03	6.5610-03	1.1180-02
8.500 03	4.3390-03	6.2230-03	1.0610-02
9.000 03	4.1800-03	5.9200-03	1.0100-02
9.500 03	3.9920-03	5.6470-03	9.6390-03
1.000 04	3.8200-03	5.4000-03	9.2200-03



TABLE 23-STOPPING POWER OF GALLIUM ARSENIDE FOR ELECTRONS

ELECTRON ENERGY EV	STOPPING POWER IN UNITS OF EV/A		
	INNER SH.	VALENCE	TOTAL
1.000 00	0.0	0.0	0.0
1.500 00	0.0	0.0	0.0
2.000 00	0.0	1.001D-02	1.001D-02
2.500 00	0.0	1.381D-02	1.381D-02
3.000 00	0.0	1.876D-02	1.876D-02
3.500 00	0.0	2.406D-02	2.406D-02
4.000 00	0.0	3.132D-02	3.132D-02
4.500 00	0.0	3.858D-02	3.858D-02
5.000 00	0.0	4.659D-02	4.659D-02
5.500 00	0.0	5.450D-02	5.450D-02
6.000 00	0.0	6.311D-02	6.311D-02
6.500 00	0.0	7.243D-02	7.243D-02
7.000 00	0.0	8.249D-02	8.249D-02
7.500 00	0.0	9.329D-02	9.329D-02
8.000 00	0.0	1.049D-01	1.049D-01
8.500 00	0.0	1.173D-01	1.173D-01
9.000 00	0.0	1.305D-01	1.305D-01
9.500 00	0.0	1.445D-01	1.445D-01
1.000 01	0.0	1.594D-01	1.594D-01
1.500 01	0.0	3.602D-01	3.602D-01
2.000 01	0.0	6.705D-01	6.705D-01
2.500 01	0.0	1.124D 00	1.124D 00
3.000 01	1.903D-01	1.672D 00	1.862D 00
3.500 01	5.295D-01	2.209D 00	2.738D 00
4.000 01	9.526D-01	2.678D 00	3.631D 00
4.500 01	1.301D 00	3.055D 00	4.356D 00
5.000 01	1.582D 00	3.346D 00	4.928D 00
5.500 01	1.889D 00	3.556D 00	5.445D 00
6.000 01	2.129D 00	3.683D 00	5.812D 00
6.500 01	2.355D 00	3.768D 00	6.132D 00
7.000 01	2.589D 00	3.811D 00	6.400D 00
7.500 01	2.776D 00	3.825D 00	6.601D 00
8.000 01	2.931D 00	3.823D 00	6.754D 00
8.500 01	3.065D 00	3.802D 00	6.867D 00
9.000 01	3.174D 00	3.771D 00	6.945D 00
9.500 01	3.261D 00	3.735D 00	6.996D 00
1.000 02	3.333D 00	3.694D 00	7.027D 00
1.500 02	3.680D 00	3.198D 00	6.879D 00
2.000 02	3.761D 00	2.772D 00	6.533D 00
2.500 02	3.727D 00	2.443D 00	6.170D 00
3.000 02	3.631D 00	2.187D 00	5.818D 00
3.500 02	3.507D 00	1.983D 00	5.491D 00
4.000 02	3.375D 00	1.817D 00	5.193D 00
4.500 02	3.244D 00	1.679D 00	4.923D 00
5.000 02	3.118D 00	1.562D 00	4.680D 00
5.500 02	2.998D 00	1.462D 00	4.460D 00
6.000 02	2.887D 00	1.375D 00	4.262D 00
6.500 02	2.783D 00	1.299D 00	4.081D 00
7.000 02	2.686D 00	1.231D 00	3.917D 00
7.500 02	2.596D 00	1.171D 00	3.767D 00
8.000 02	2.512D 00	1.117D 00	3.629D 00
8.500 02	2.434D 00	1.069D 00	3.502D 00
9.000 02	2.360D 00	1.024D 00	3.385D 00
9.500 02	2.292D 00	9.838D-01	3.276D 00
1.000 03	2.228D 00	9.467D-01	3.174D 00
1.500 03	1.769D 00	6.952D-01	2.464D 00
2.000 03	1.504D 00	5.553D-01	2.059D 00
2.500 03	1.322D 00	4.652D-01	1.787D 00
3.000 03	1.185D 00	4.020D-01	1.587D 00
3.500 03	1.077D 00	3.549D-01	1.432D 00
4.000 03	9.894D-01	3.184D-01	1.308D 00
4.500 03	9.166D-01	2.891D-01	1.206D 00
5.000 03	8.550D-01	2.651D-01	1.120D 00
5.500 03	8.021D-01	2.451D-01	1.047D 00
6.000 03	7.560D-01	2.281D-01	9.841D-01
6.500 03	7.156D-01	2.134D-01	9.290D-01
7.000 03	6.797D-01	2.006D-01	8.804D-01
7.500 03	6.477D-01	1.894D-01	8.371D-01
8.000 03	6.189D-01	1.795D-01	7.983D-01
8.500 03	5.928D-01	1.706D-01	7.634D-01
9.000 03	5.691D-01	1.626D-01	7.316D-01
9.500 03	5.474D-01	1.554D-01	7.027D-01
1.000 04	5.275D-01	1.488D-01	6.762D-01

TABLE 2C-CSDA RANGE AND STRAGGLING OF ELECTRONS IN GALLIUM ARSENIDE

ELECTRON ENERGY EV	CSDA RANGE (E - 10EV) A	MEAN SQ. EN. LOSS EV/A	MEAN SQ. RANGE FL. A2	RELATIVE RANGE STRAGGLING
1.500 01	2.1000 01	7.8910-01	5.1530 02	1.0810 00
2.000 01	3.1230 01	2.3670 00	6.7240 02	8.3040-01
2.500 01	3.7050 01	5.5990 00	7.3480 02	7.3170-01
3.000 01	4.0500 01	1.1560 01	7.6360 02	6.8240-01
3.500 01	4.2700 01	2.6120 01	7.7890 02	6.5360-01
4.000 01	4.4230 01	4.7690 01	7.8830 02	6.3410-01
4.500 01	4.5530 01	7.3040 01	7.9490 02	6.1920-01
5.000 01	4.5610 01	9.5590 01	8.0020 02	6.0690-01
5.500 01	4.7570 01	1.1520 02	8.0460 02	5.9630-01
6.000 01	4.8460 01	1.3620 02	8.0870 02	5.8690-01
6.500 01	4.9290 01	1.5330 02	8.1250 02	5.7820-01
7.000 01	5.0090 01	1.7020 02	8.1610 02	5.7030-01
7.500 01	5.0860 01	1.8630 02	8.1960 02	5.6290-01
8.000 01	5.1610 01	2.0060 02	8.2310 02	5.5590-01
8.500 01	5.2340 01	2.1310 02	8.2650 02	5.4930-01
9.000 01	5.3070 01	2.2440 02	8.3000 02	5.4290-01
9.500 01	5.3780 01	2.3430 02	8.3350 02	5.3680-01
1.000 02	5.4500 01	2.4290 02	8.3710 02	5.3090-01
1.500 02	6.1630 01	2.5060 02	8.7830 02	4.8080-01
2.000 02	6.9080 01	3.1020 02	9.3380 02	4.4230-01
2.500 02	7.6960 01	3.5710 02	1.0070 03	4.1240-01
3.000 02	8.5310 01	3.9210 02	1.1020 03	3.8910-01
3.500 02	9.4160 01	4.1590 02	1.2190 03	3.7090-01
4.000 02	1.0350 02	4.3140 02	1.3630 03	3.5660-01
4.500 02	1.1340 02	4.4120 02	1.5350 03	3.4550-01
5.000 02	1.2380 02	4.4710 02	1.7390 03	3.3670-01
5.500 02	1.3480 02	4.5060 02	1.9760 03	3.2980-01
6.000 02	1.4630 02	4.5240 02	2.2490 03	3.2430-01
6.500 02	1.5420 02	4.5230 02	2.5620 03	3.1980-01
7.000 02	1.7080 02	4.5250 02	2.9160 03	3.1620-01
7.500 02	1.8380 02	4.5160 02	3.3140 03	3.1320-01
8.000 02	1.9730 02	4.5020 02	3.7590 03	3.1070-01
8.500 02	2.1130 02	4.4860 02	4.2530 03	3.0860-01
9.000 02	2.2590 02	4.4670 02	4.8000 03	3.0680-01
9.500 02	2.4090 02	4.4470 02	5.4010 03	3.0510-01
1.000 03	2.5640 02	4.4270 02	6.0600 03	3.0360-01
1.500 03	4.3700 02	4.4060 02	1.6550 04	2.9440-01
2.000 03	6.6030 02	4.3990 02	3.6870 04	2.9080-01
2.500 03	9.2180 02	4.6630 02	7.1280 04	2.8960-01
3.000 03	1.2190 03	4.8840 02	1.2380 05	2.8850-01
3.500 03	1.5520 03	5.0250 02	1.9850 05	2.8710-01
4.000 03	1.9180 03	5.1140 02	2.9940 05	2.8540-01
4.500 03	2.3160 03	5.1650 02	4.3080 05	2.8340-01
5.000 03	2.7470 03	5.1920 02	5.9700 05	2.8130-01
5.500 03	3.2090 03	5.2010 02	8.0240 05	2.7920-01
6.000 03	3.7010 03	5.2010 02	1.0510 06	2.7700-01
6.500 03	4.2250 03	5.1930 02	1.3490 06	2.7490-01
7.000 03	4.7780 03	5.1810 02	1.6990 06	2.7280-01
7.500 03	5.3600 03	5.1660 02	2.1080 06	2.7080-01
8.000 03	5.9720 03	5.1480 02	2.5790 06	2.6890-01
8.500 03	6.6130 03	5.1290 02	3.1170 06	2.6700-01
9.000 03	7.2820 03	5.1090 02	3.7290 06	2.6520-01
9.500 03	7.9300 03	5.0880 02	4.4180 06	2.6340-01
1.000 04	8.7050 03	5.0680 02	5.1910 06	2.6170-01

TABLE 3-UNIVERSAL FUNCTIONS EMPLOYED IN THE EVALUATION OF INVER  
SHELL CONTRIBUTIONS TO INVERSE MEAN FREE PATH, STOPPING  
POWER, AND MEAN SQUARE ENERGY LOSS (SEE DEFINITIONS AND  
DETAILS IN SECTION IV)

BETA	F	G	H
1.200 00	2.2790-01	2.3930-01	2.5160-01
1.400 00	4.8730-01	5.2730-01	5.8060-01
1.700 00	7.4970-01	8.7220-01	1.0220 00
2.000 00	9.0540-01	1.1060 00	1.3700 00
2.500 00	1.0170 00	1.3260 00	1.7760 00
3.000 00	1.0370 00	1.4240 00	2.0340 00
3.500 00	1.0190 00	1.4580 00	2.2020 00
4.000 00	9.8580-01	1.4590 00	2.3130 00
5.000 00	9.0770-01	1.4160 00	2.4350 00
6.000 00	8.3310-01	1.3510 00	2.4850 00
7.000 00	7.6770-01	1.2840 00	2.5000 00
8.000 00	7.1130-01	1.2200 00	2.4960 00
9.000 00	6.6260-01	1.1600 00	2.4830 00
1.000 01	6.2050-01	1.1060 00	2.4640 00
1.300 01	5.2240-01	9.7020-01	2.3960 00
1.600 01	4.5270-01	3.6610-01	2.3280 00
2.000 01	3.8580-01	7.6040-01	2.2460 00
2.500 01	3.2720-01	6.6280-01	2.1620 00
3.000 01	2.8500-01	5.8970-01	2.0930 00
4.000 01	2.2800-01	4.8670-01	1.9870 00
5.000 01	1.9100-01	4.1710-01	1.9110 00
6.000 01	1.6500-01	3.6650-01	1.8520 00
7.000 01	1.4550-01	3.2790-01	1.8060 00
8.000 01	1.3040-01	2.9730-01	1.7680 00
1.000 02	1.0840-01	2.5180-01	1.7100 00
1.300 02	8.7030-02	2.0630-01	1.6500 00
1.600 02	7.3020-02	1.7570-01	1.6080 00
2.000 02	6.0370-02	1.4760-01	1.5680 00
2.500 02	4.9850-02	1.2370-01	1.5330 00
3.000 02	4.2590-02	1.0690-01	1.5080 00
4.000 02	3.3180-02	8.4650-02	1.4740 00
5.000 02	2.7300-02	7.0520-02	1.4520 00